INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2005-06
Statistics - II, Backpaper Examination

(15) 1. Let $X_1, \ldots, X_n$ be independent random variables with densities

$$f_{X_i}(x_i|\theta) = \begin{cases} \frac{1}{2i\theta} & -i(\theta-1) < x_i < i(\theta+1) \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$.
(a) Find a two-dimensional sufficient statistic for θ.
(b) Find the maximum likelihood estimator of θ.

(7) 2. Suppose X_1 and X_2 are two i.i.d. observations from Binomial(n, p), $0 < p < 1$, n known. Let $\theta = p^n$. Find the UMVUE of θ.

(8) 3. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from Poisson(λ). Consider testing

$$H_0 : \lambda \leq 1 \text{ versus } H_1 : \lambda > 1.$$

(a) Show that the conditions required for the existence of a UMP test are satisfied here.
(b) Derive the UMP test of level α.

(20) 4. A large shipment of parts is received, out of which 5 are tested for defects. The number of defective parts in the sample, X is assumed to be Binomial with parameter θ. From past shipments it is known that θ has a Beta(1, 9) distribution.
(a) Find the HPD estimate of θ if $x = 0$ is observed.
(b) Find a 95% credible set for θ if $x = 0$ is observed.
(c) Consider testing $H_0 : \theta \leq 0.10$ versus $H_1 : \theta > 0.10$. Find the posterior probability of H_0 and explain what can be concluded from this.